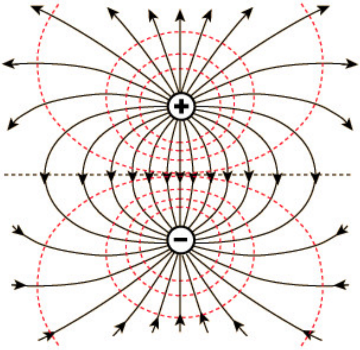
Electric Potential



"Electricity is really just organized lightning."
- *George Carlin*

David J. Starling
Penn State Hazleton
PHYS 212

Electric Potential

Since the electric force is so similar to gravity, might it be **conservative?**

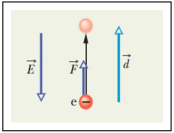
- Yes!
- If we move charges around in a field, we have:

$$\Delta U = U_f - U_i = -W \tag{1}$$

- U is the potential energy
- This work is *path independent*
- We say the energy between particles is zero when their separation is infinity (this is an arbitrary choice)

Electric Potential

If we move a charged particle a distance d in a constant $\vec{E}$-field,



- The work done is $W = \vec{F} \cdot \vec{d}$
- But the force is $\vec{F} = q\vec{E}$, so

$$W = q\vec{E} \cdot \vec{d} = qEd\cos(\theta)$$

- If the E-field changes, we have to do an integral:

$$W = \int q\vec{E} \cdot \vec{ds} = \int qE\cos(\theta)ds$$

- So $\Delta U = -\int qE\cos(\theta)ds$

Electric Potential

There is a more convenient incarnation of electric potential energy: **electric potential**

- Definition: $V = U/q$ and $\Delta V = \Delta U/q$.
- This removes the influence of the "test charge"
- Think of the connection:

$$\vec{E} = \vec{F}/q \quad \text{(vector relationship)} \tag{2}$$

$$V = U/q \quad \text{(scalar relationship)} \tag{3}$$

- Units of Potential: J/C = N m/C = V $\rightarrow$ volt
- Units of Electric Field: N/C = V/m

Electric Potential

Question: A negative charge sits at the origin. Where is the electric potential (V) minimum?

(a) At the origin
(b) At infinity
(c) Somewhere in between
(d) Can't determine

Electric Potential

Main Idea: Just like *potential energy*, the electric potential is a guide to where a positive test particle will go if set free.

Lecture Question 7.1: A *positive* charge sits at the origin. Where is the electric potential (V) minimum?

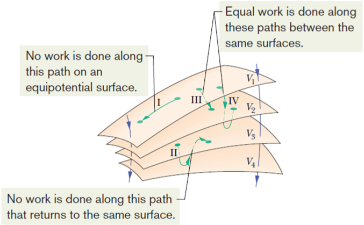
- **(a)** At the origin
- **(b)** At infinity
- **(c)** Somewhere in between
- **(d)** Can’t determine

Equipotential Surfaces

- If we bring in a charge from infinity, it takes work
- The surface on which this work is the same is called an

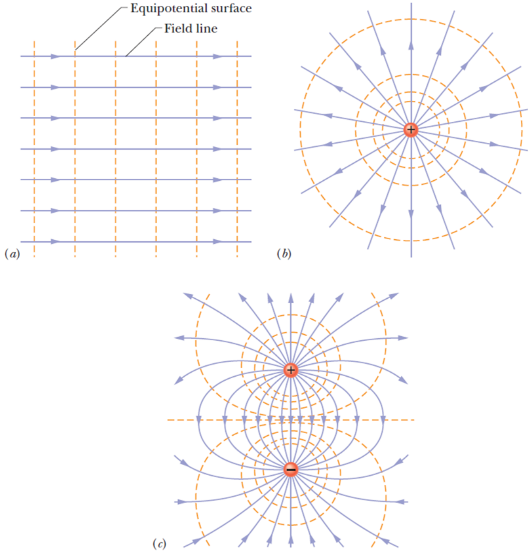
Equipotential Surface

- Moving the charge along this surface takes no work



- $\vec{E}$ is always $\perp$ to the equipotential surfaces

Equipotential Surfaces



Equipotential Surfaces

What about the surface of a conductor?

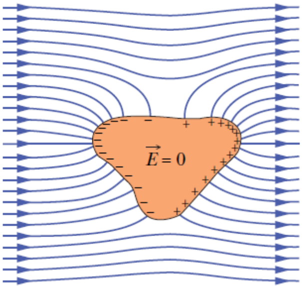


Fig. 24-20 An uncharged conductor is suspended in an external electric field. The free electrons in the conductor distribute themselves on the surface as shown, so as to reduce the net electric field inside the conductor to zero and make the net field at the surface perpendicular to the surface.

Potential from Field, and vice versa

Let's say we know the field everywhere: $\vec{E}(\vec{r})$. Can we find the potential?

- $\Delta V = \Delta U/q = -W/q$ (definition of potential)
- So $\Delta V = -\vec{F} \cdot \vec{d}/q = -(q\vec{E}) \cdot \vec{d}/q = -\vec{E} \cdot \vec{d}$
- Or, if the field varies, we do an integral:

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} \tag{4}$$

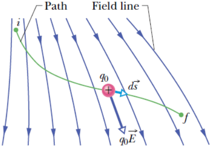


Fig. 24-4 A test charge q_0 moves from point i to point f along the path shown in a nonuniform electric field. During a displacement $d\vec{s}$, an electrostatic force $q_0\vec{E}$ acts on the test charge. This force points in the direction of the field line at the location of the test charge.

The potential difference between two points $\vec{r}_i$ and $\vec{r}_f$ for a field $\vec{E}$ is the line integral of $\vec{E}$ between those two points *along any path*.

Potential from Field, and vice versa

What if we know the potential $V(\vec{r})$, can we find $\vec{E}(\vec{r})$?

- Let's just look in 1D for a moment:

$$V(x_f) - V(x_i) = \Delta V_x = -\int_{x_i}^{x_f} E_x dx \tag{5}$$

$$\text{or} \tag{6}$$

$$V(x) = -\int E_x dx \tag{7}$$

- But the derivative is the inverse of the integral, so let's differentiate both sides:

$$\frac{dV}{dx} = -E_x \tag{8}$$

Potential from Field, and vice versa

What if we know the potential $V(\vec{r})$, can we find $\vec{E}(\vec{r})$?

- In general:

$$E_x = -\frac{dV}{dx} \tag{9}$$

$$E_y = -\frac{dV}{dy} \tag{10}$$

$$E_z = -\frac{dV}{dz} \tag{11}$$

- Or, we can write it like this:

$$\vec{E}(\vec{r}) = -\frac{dV}{dx}\hat{i} - \frac{dV}{dy}\hat{j} - \frac{dV}{dz}\hat{k} \tag{12}$$

Potential from Point Charges

The potential from a single point charge is given by

$$V_q = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \tag{13}$$

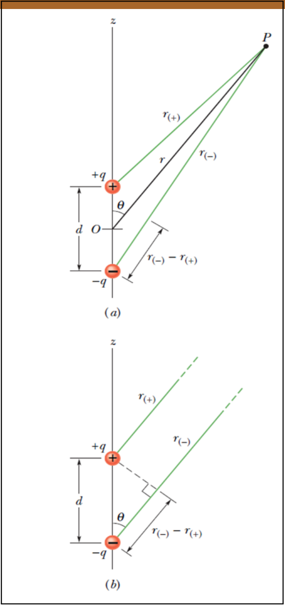
This comes from integrating $E \propto 1/r^2$ from infinity (where $V = 0$) down to r.

Potential from Point Charges

Find the electric potential from a dipole at point P, far away from the charges.

$$V_p = V_+ + V_- = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_+} + \frac{-q}{r_-}\right)$$
$$V_p = \frac{q}{4\pi\epsilon_0}\left(\frac{r_- - r_+}{r_- r_+}\right)$$
$$V_p \approx \frac{q}{4\pi\epsilon_0}\left(\frac{d\cos(\theta)}{r^2}\right)$$
$$V_p \approx \frac{1}{4\pi\epsilon_0}\frac{p\cos(\theta)}{r^2}$$

Compare this to $E_p = \frac{1}{2\pi\epsilon_0}\frac{p}{z^3}$.



Lecture Question 7.3: Two equal and opposite charges $\pm q$ sit on either side of the origin on the x-axis. If they are placed at $x = \pm d/2$, what is the potential at the origin?

(a) 0

(b) 1

(c) $\frac{1}{2\pi\epsilon_0}\frac{p}{z^3}$

(d) Can't determine