Chapter 4 - Motion in 2D and 3D

"Never confuse motion with action."
- Benjamin Franklin

David J. Starling
Penn State Hazleton

PHYS 211

Generalize to 3D

Position, displacement, velocity and acceleration can be generalized to 3D using vectors.

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v(t) \rightarrow \qquad \vec{v}(t) &= \frac{d\vec{r}}{dt} = v_x(t)\hat{i} + v_y(t)\hat{j} + v_z(t)\hat{k} \\
a_{avg}(t) \rightarrow \qquad \vec{a}_{avg}(t) &= \frac{\Delta\vec{v}}{\Delta t}
\end{aligned}
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$$

Generalize to 3D

We can also generalize two of our constant acceleration equations.

$$v(t) = v_0 + at \qquad \rightarrow \vec{v}(t) = \vec{v}_0 + \vec{a}t$$

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2 \qquad \rightarrow \vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

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$$\rightarrow v_x^2 = v_{0,x}^2 + 2a_x \Delta x$$

$$\rightarrow v_y^2 = v_{0,y}^2 + 2a_y \Delta y$$

$$\rightarrow v_z^2 = v_{0,z}^2 + 2a_z \Delta z$$

Objectives (Ch 4)

Lecture Question 4.1

When an object is thrown (ignoring air drag), after it has left the thrower's hand,

(a) v_x and v_y are constant.

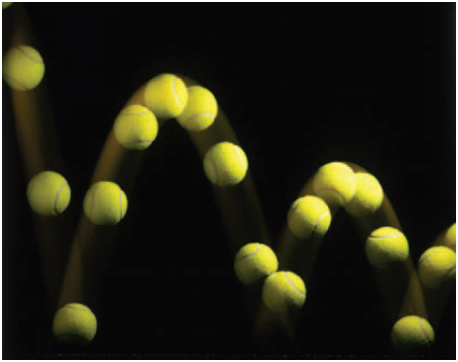
(b) v_x and v_y change with time.

(c) v_x changes with time but v_y is constant.

(d) v_x is constant but v_y changes with time.

Projectile Motion

Projectile motion is a very common example of 2D motion where objects move under the influence of gravity.



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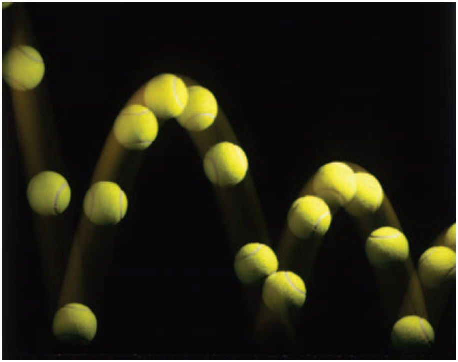
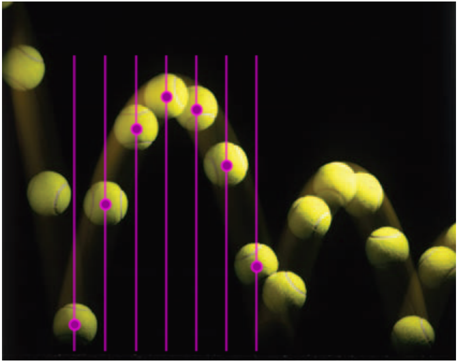


This ball is also rotating — we'll get to that later (Ch 10).

Projectile Motion

In projectile motion, the acceleration in the **horizontal direction** *is* 0 m/s^2.



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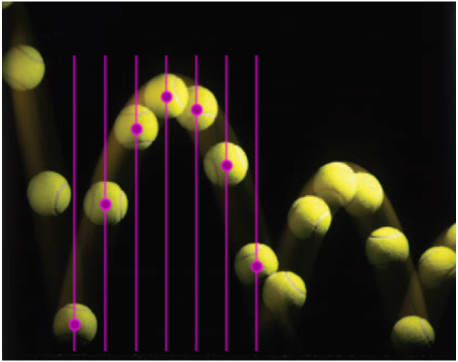
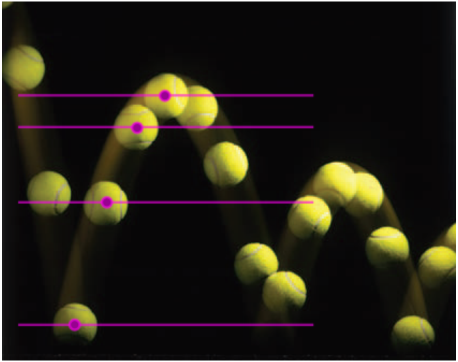


If we pick $+x$ as right, $a_x = 0\ \mathrm{m/s^2}$.

Projectile Motion

In projectile motion, the acceleration in the **vertical direction** *is* $g = 9.81\ m/s^2$.



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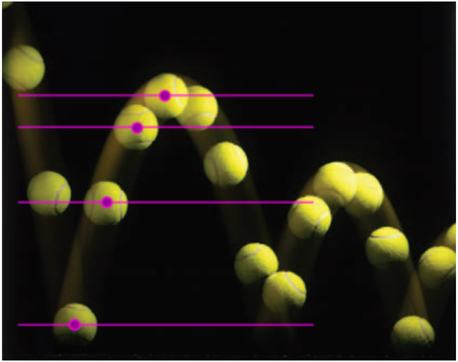
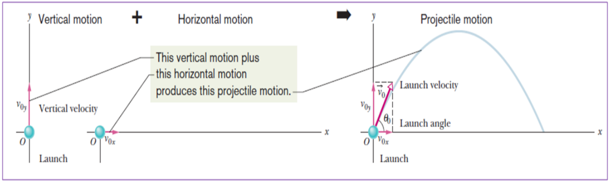


If we pick $+y$ as up, $a_y = -9.8\ \text{m/s}^2$.

Projectile Motion

In projectile motion, the horizontal and vertical motion are independent of each other.



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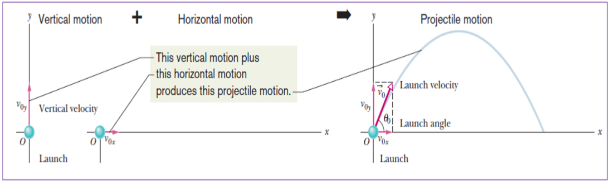


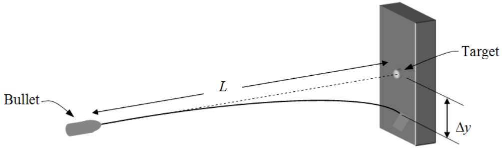
We use our standard equations:

$$
\begin{aligned}
x(t) &= x_0 + v_{0,x}t + \frac{1}{2}a_x t^2 \\
y(t) &= y_0 + v_{0,y}t + \frac{1}{2}a_y t^2
\end{aligned}
$$

Projectile Motion

Lecture Question 4.2

A bullet is aimed at a target on the wall a distance L away from the firing position and the bullet strikes the wall a distance Δy below the mark. If the distance L was half as large, and the bullet had the same initial velocity, how would Δy change?



(a) $\Delta y \to 2\Delta y$

(b) $\Delta y \to 4\Delta y$

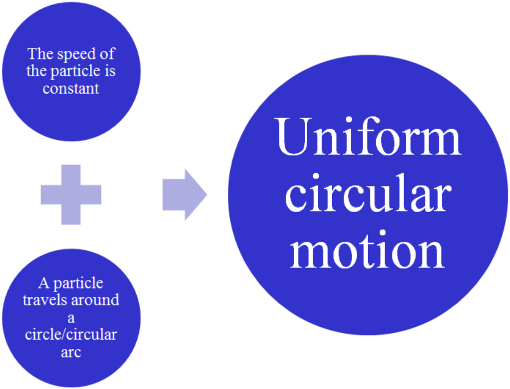
(c) $\Delta y \to \Delta y/2$

(d) $\Delta y \to \Delta y/4$

(e) Need more information.

Uniform Circular Motion

An object is in **uniform circular motion** *when its speed is constant and it travels in a circle.*



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An object moving in a circle experiences acceleration (even if it's moving at constant speed!).

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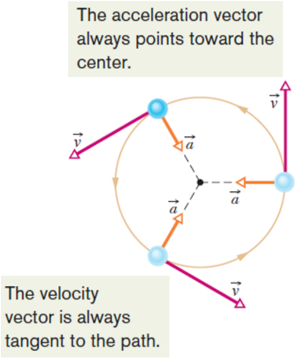

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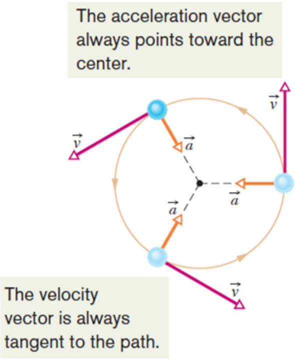

If the object moves faster, should the acceleration be larger or smaller?

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For uniform circular motion, we can find the centripetal acceleration a_r *using geometry and calculus.*

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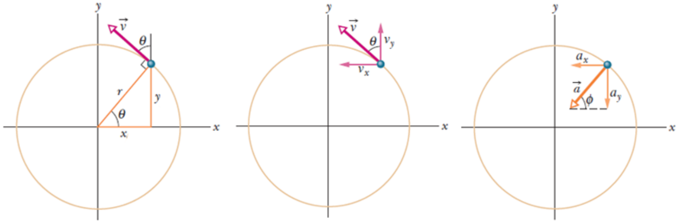

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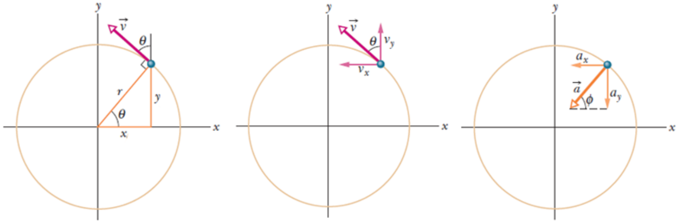

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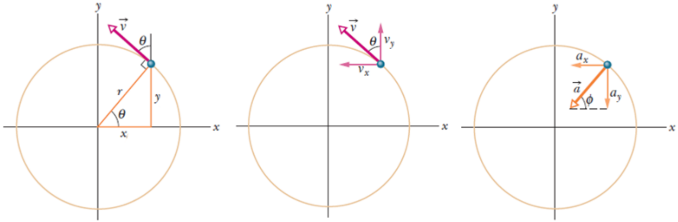

$$\begin{aligned}\vec{v} &= v_x\hat{i} + v_y\hat{j} \\ &= [-v\sin(\theta)]\hat{i} + [v\cos(\theta)]\hat{j}\end{aligned}$$

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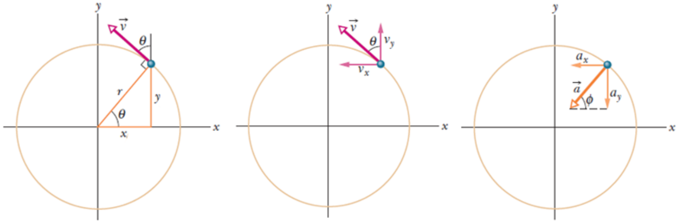

$$\begin{aligned}\vec{v} &= v_x\hat{i} + v_y\hat{j} \\ &= [-v\sin(\theta)]\hat{i} + [v\cos(\theta)]\hat{j} \\ &= \left(-\frac{vy}{r}\right)\hat{i} + \left(\frac{vx}{r}\right)\hat{j}\end{aligned}$$

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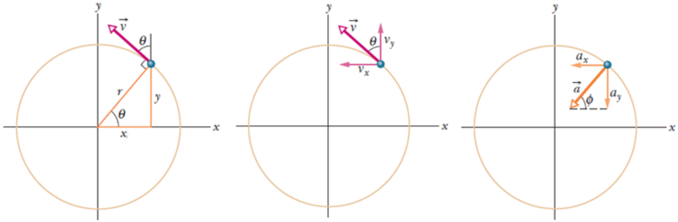

$$
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\vec{v} &= v_x\hat{i} + v_y\hat{j} \\
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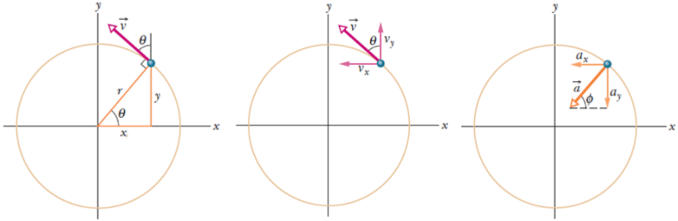
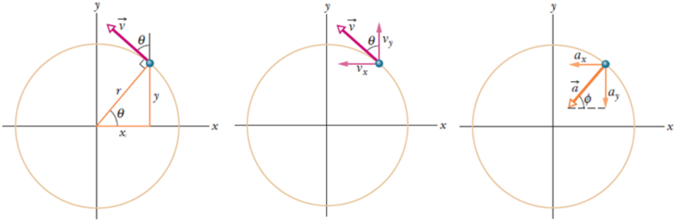
&= \left(-\frac{vy}{r}\right)\hat{i} + \left(\frac{vx}{r}\right)\hat{j} \\
\vec{a} &= \frac{d\vec{v}}{dt}
\end{aligned}
$$

Uniform Circular Motion



$$\vec{v} = \frac{v}{r}\left(-y\hat{i} + x\hat{j}\right)$$

Uniform Circular Motion



$$\vec{v} = \frac{v}{r}\left(-y\hat{i} + x\hat{j}\right)$$

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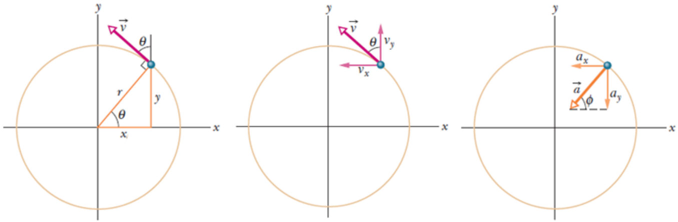
Uniform Circular Motion

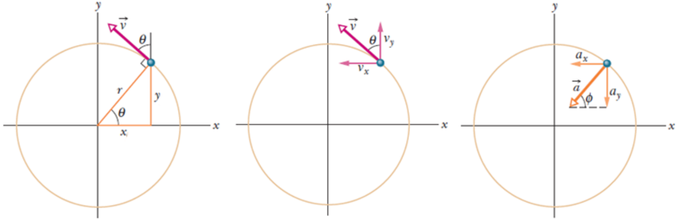


$$\vec{v} = \frac{v}{r}\left(-y\hat{i} + x\hat{j}\right)$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{v}{r}\left(-v_y\hat{i} + v_x\hat{j}\right)$$

$$a = \sqrt{a_x^2 + a_y^2}$$

Uniform Circular Motion



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\vec{v} &= \frac{v}{r}\left(-y\hat{i} + x\hat{j}\right) \\
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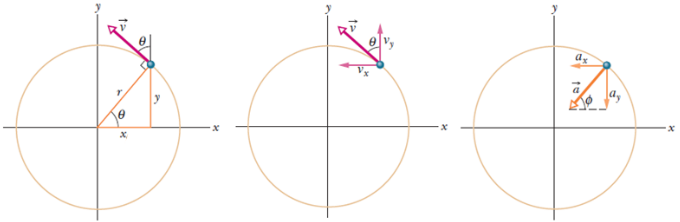
a &= \sqrt{a_x^2 + a_y^2} \\
&= \frac{v}{r}\sqrt{v_y^2 + v_x^2}
\end{aligned}
$$

Uniform Circular Motion



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$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{v}{r}\left(-v_y\hat{i} + v_x\hat{j}\right)$$

$$a = \sqrt{a_x^2 + a_y^2}$$

$$= \frac{v}{r}\sqrt{v_y^2 + v_x^2}$$

$$a = \frac{v^2}{r} \text{ (uniform circular motion)}$$

Uniform Circular Motion

Lecture Question 4.3

A steel ball is whirled on the end of a chain in a horizontal circle of radius R with a constant period T. If the radius of the circle is then reduced to $0.75R$, while the period remains T, what happens to the centripetal acceleration of the ball?

(a) Centripetal acceleration increases.

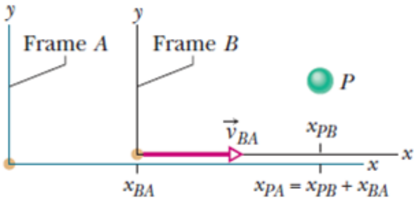
(b) Centripetal acceleration decrease.

(c) Centripetal acceleration stays the same.

(d) Not enough information.

Relative Motion

The velocity of an object depends on the reference frame from which it is measured.



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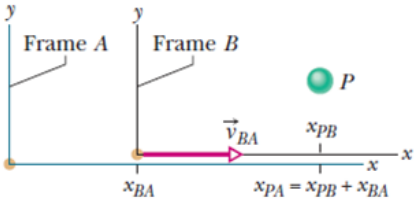
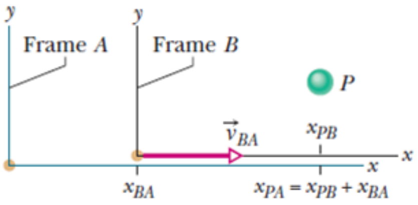


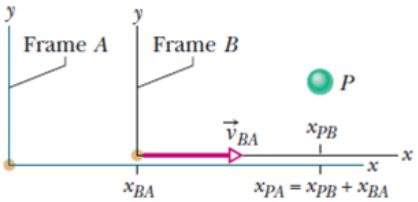
- frame A (Alice) is stationary
- frame B (Bob) moves with some constant velocity
- object P (Parakeet) is measured

Relative Motion



- x_{BA}: position of Bob relative to Alice
- x_{PB}: position of Parakeet relative to Bob
- x_{PA}: position of Parakeet relative to Alice

Relative Motion



- x_{BA}: position of Bob relative to Alice
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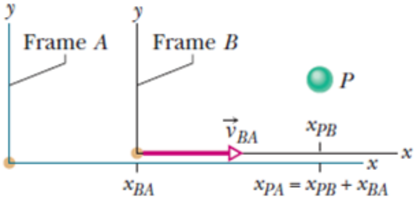
$$x_{PA} = x_{PB} + x_{BA}$$

Relative Motion



- x_{BA}: position of Bob relative to Alice
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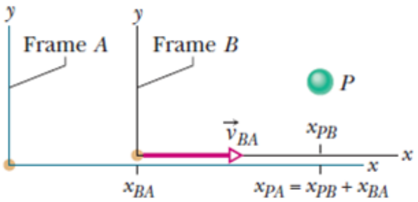
$$
\begin{aligned}
x_{PA} &= x_{PB} + x_{BA} \\
v_{PA} &= v_{PB} + v_{BA}
\end{aligned}
$$

Relative Motion



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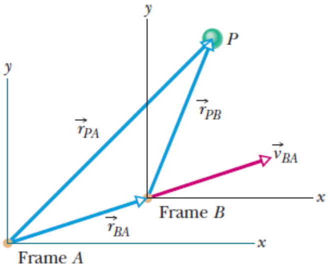
$$\begin{aligned} x_{PA} &= x_{PB} + x_{BA} \\ v_{PA} &= v_{PB} + v_{BA} \\ a_{PA} &= a_{PB} + a_{BA} \end{aligned}$$

Relative Motion



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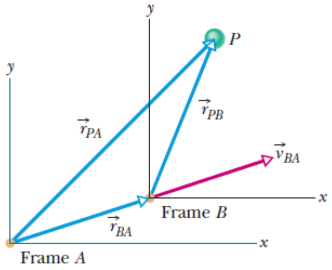

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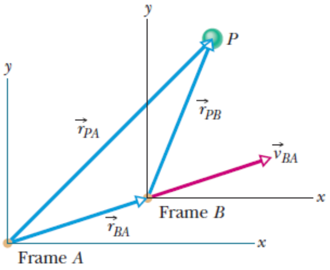
$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

Relative Motion



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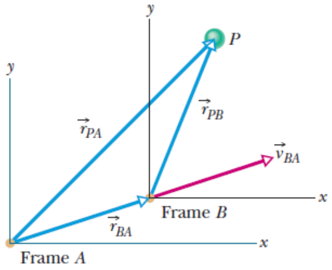
Relative Motion



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$$\begin{aligned} \vec{r}_{PA} &= \vec{r}_{PB} + \vec{r}_{BA} \\ \vec{v}_{PA} &= \vec{v}_{PB} + \vec{v}_{BA} \\ \vec{a}_{PA} &= \vec{a}_{PB} + \vec{a}_{BA} \end{aligned}$$