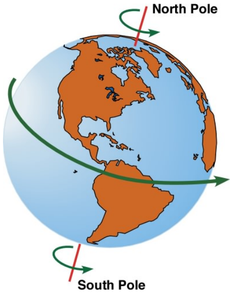
Chapter 10 - Rotation



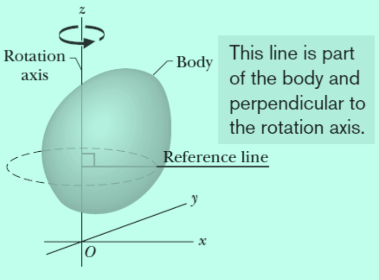
"To know that we know what we know, and to know that we do not know what we do not know, that is true knowledge."

-Nicolas Copernicus

David J. Starling
Penn State Hazleton
PHYS 211

Rotational Variables

We want to describe the rotation of a **rigid body** *about a* **fixed axis**.



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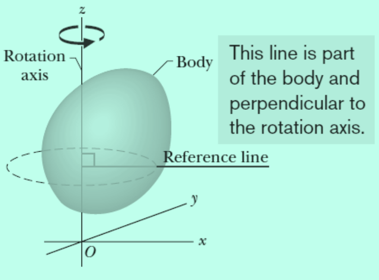
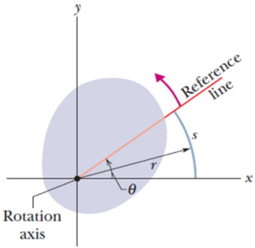


The object does not deform, and the axis stays put.

Rotational Variables

The kinematic and dynamic variables we have used so far have their rotational counterparts:



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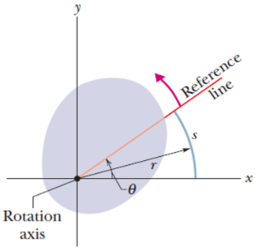

How far has the rigid body rotated?

$$\theta = \frac{s}{r} \quad \text{and} \quad \theta \leftrightarrow x$$

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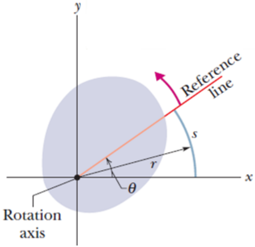


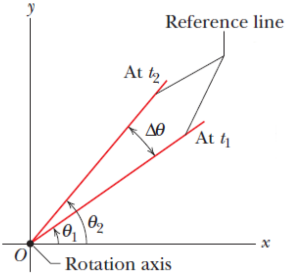
How far has the rigid body rotated?

$$\theta = \frac{s}{r} \quad \text{and} \quad \theta \leftrightarrow x$$

[note: 1 revolution = 360° = 2π radians]

Rotational Variables

If θ is like position, then $\Delta\theta$ is like displacement.



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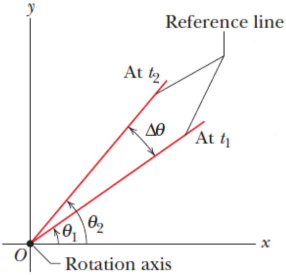

This is the *reference line* of a rigid body.

$$\Delta\theta = \theta_2 - \theta_1$$

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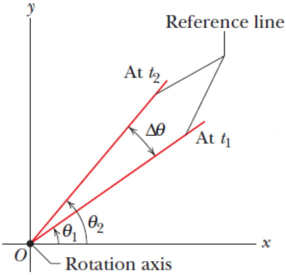


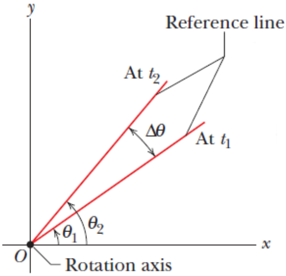
This is the *reference line* of a rigid body.

$$\Delta\theta = \theta_2 - \theta_1$$

[note: counterclockwise is positive]

Rotational Variables

Angular velocity ω *is like linear velocity, except* $x \to \theta$.



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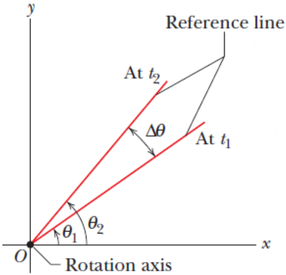
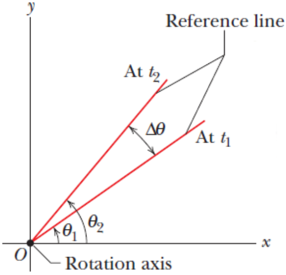


$$\omega_{avg} = \frac{\Delta\theta}{\Delta t} \quad \text{and} \quad \omega = \frac{d\theta}{dt}$$

Rotational Variables

Angular acceleration α *is like linear acceleration, except* $v \rightarrow \omega$.



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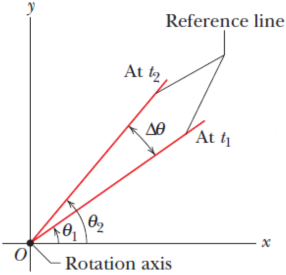

$$\alpha_{avg} = \frac{\Delta\omega}{\Delta t} \quad \text{and} \quad \alpha = \frac{d\omega}{dt}$$

Rotational Variables

The linear variables have their rotational counterparts:

linear	angular		units
x	θ		rad
v	ω	$d\theta/dt$	rad/s
a	α	$d\omega/dt$	rad/s^2

Rotational Variables

The linear variables have their rotational counterparts:

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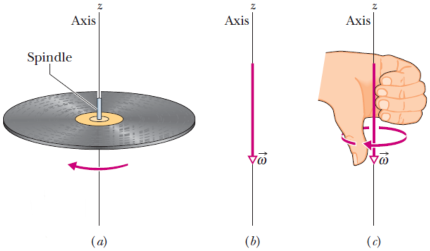
We use the right-hand rule to assign direction:



Rotational Variables

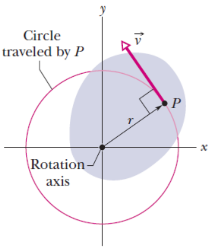
On a rigid body,

- all points trace out a circle;
- all points have the *same angular displacement*

  $\Delta\theta = \Delta s / r$

- all points have the *same angular velocity*

$$\omega = \frac{d\theta}{dt} = \frac{d(s/r)}{dt} = \frac{v_t}{r}$$

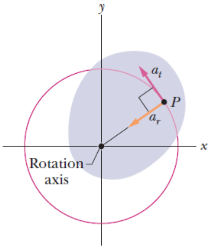


(a)

Rotational Variables

Taking another derivative,

$$\alpha = \frac{d\omega}{dt} = \frac{d(v_t/r)}{dt} = \frac{a_t}{r}$$



(b)

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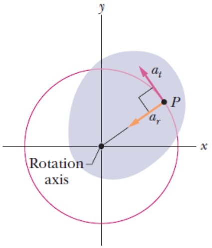


(b)

$$\begin{aligned} s &= r\theta \\ v_t &= r\omega \\ a_t &= r\alpha \\ a_r &= v^2/r = \omega^2 r \end{aligned}$$

Rotational Variables

If the angular acceleration of a rotating object is constant, we can derive the same **constant acceleration equations** *as before.*

linear	angular
$v(t) = v_0 + at$	$\omega(t) = \omega_0 + \alpha t$
$x(t) = x_0 + v_0 t + 0.5at^2$	$\theta(t) = \theta_0 + \omega_0 t + 0.5\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Rotational Variables

Lecture Question 10.1

The Earth, which has an equatorial radius of 6380 km, makes one revolution on its axis every 23.93 hours. What is the tangential speed of Nairobi, Kenya, a city near the equator?

(a) 37.0 m/s

(b) 74.0 m/s

(c) 148 m/s

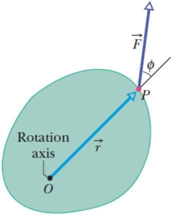
(d) 232 m/s

(e) 465 m/s

Torque

Rotational motion is generated by torque:

$$\tau = rF \sin \phi.$$



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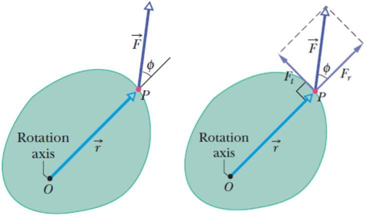

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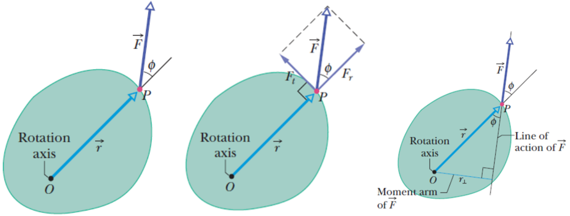

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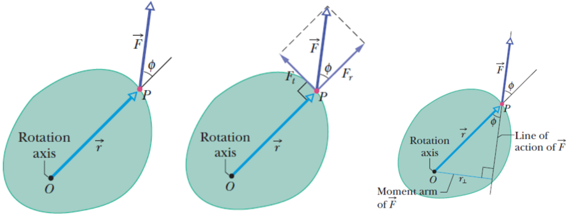

- $\tau = (r)(F\sin\phi) = rF_t$
- $\tau = (r\sin\phi)(F) = r_\perp F$

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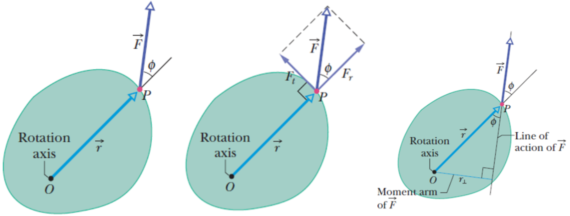
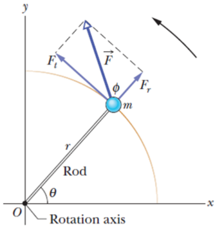


- $\tau = (r)(F\sin\phi) = rF_t$
- $\tau = (r\sin\phi)(F) = r_\perp F$
- $r_\perp = r\sin(\phi)$ is the moment arm

Torque

When a torque is applied to an object, it accelerates angularly.



$$F_t = ma_t$$

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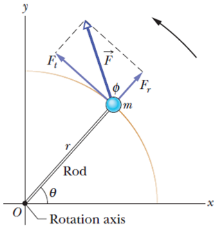

$$F_t = ma_t$$
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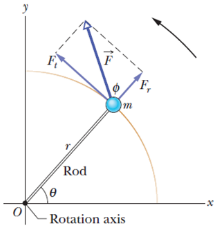

$$\begin{aligned} F_t &= ma_t \\ \tau = F_t r &= ma_t r \\ \tau &= m(r\alpha)r \end{aligned}$$

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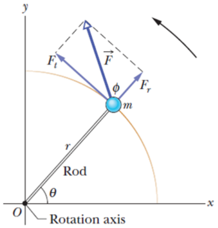

$$
\begin{aligned}
F_t &= ma_t \\
\tau = F_t r &= ma_t r \\
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$$

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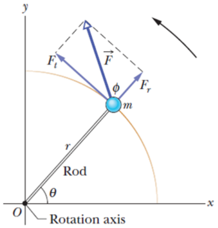

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\tau &= (mr^2)\alpha \\
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\end{aligned}
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Moment of Inertia

When torque is applied to an object, it resists this motion. For a point particle, we found

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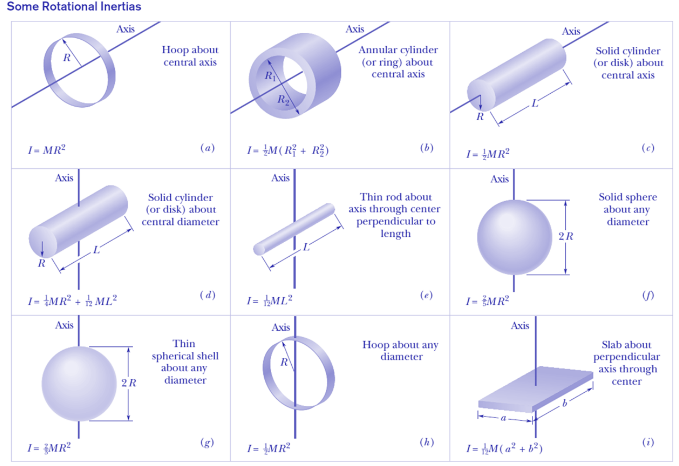
For many point particles in a rigid body, we sum them up:

$$I = \sum_i m_i r_i^2 \rightarrow \int r^2 dm.$$

Moment of Inertia

Moment of Inertia for common shapes:

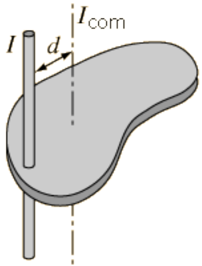


Moment of Inertia

If I_{com} is known for an axis through the center of mass, then **any parallel axis** *has*

$$I = I_{com} + Md^2$$

a distance d away.



Rotational Energy

To find the kinetic energy of a rotating object, split it up into small masses:

$$K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \ldots$$

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&= \sum_i \frac{1}{2}m_iv_i^2 \\
&= \sum_i \frac{1}{2}m_i(r_i\omega)^2 \\
&= \sum_i \frac{1}{2}m_ir_i^2\omega^2 \\
&= \frac{1}{2}\left(\sum_i m_ir_i^2\right)\omega^2 \\
K &= \frac{1}{2}I\omega^2
\end{aligned}$$

Rotational Energy

We can change the kinetic energy by doing work ($W = \Delta K$, right?):

$$W = \int_{x_i}^{x_f} F\,dx \rightarrow W = \int_{\theta_i}^{\theta_f} \tau\,d\theta$$

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[Constant torque: $W = \tau(\theta_f - \theta_i)$.]

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Power is just the derivative of work, so

$$P = \frac{dW}{dt} = \tau\omega.$$

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[Constant torque: $W = \tau(\theta_f - \theta_i)$.]

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[compare: $P = Fv$]

Rotational Energy

TABLE 10-3

Some Corresponding Relations for Translational and Rotational Motion

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Position	x	Angular position	θ
Velocity	$v = dx/dt$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F_{net} = ma$	Newton's second law	$\tau_{net} = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

Rotational Energy

Lecture 10.4

Two solid cylinders are rotating about an axis that passes through the center of both ends of each cylinder. Cylinder A has three times the mass and twice the radius of cylinder B, but they have the same rotational kinetic energy. What is the ratio of the angular velocities, ω_A/ω_B, for these two cylinders?

(a) 0.29

(b) 0.50

(c) 1.0

(d) 2.0

(e) 4.0