Practice Exam #3

Name: ______________________________

Useful Equations

$$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$
$$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$
$$v_x(t) = v_{0x} + a_x t$$
$$v_y(t) = v_{0y} + a_y t$$
$$v_{fx}^2 = v_{0x}^2 + 2a_x \Delta x$$
$$v_{fy}^2 = v_{0y}^2 + 2a_y \Delta y$$

$$a_c = \frac{v^2}{r}$$
$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega(t) = \omega_0 + \alpha t$$
$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$
$$\sum_i \vec{\tau}_i = I\vec{\alpha} = \frac{d\vec{L}}{dt}$$

$$x(t) = A\cos(\omega t + \phi_0)$$
$$\omega = 2\pi f = 2\pi/T$$
$$v_{max} = A\omega$$
$$a_{max} = A\omega^2$$
$$v = \sqrt{F_T/\mu}$$
$$v = \lambda f$$
$$\omega_{spring} = \sqrt{k/m}$$
$$\omega_{pendulum} = \sqrt{g/L}$$
$$k = 2\pi/\lambda$$

$$\sum_i \vec{F}_i = m\vec{a} = \frac{d\vec{p}}{dt}$$
$$\vec{p} = m\vec{v}$$
$$F_{fr} = \mu_{s,k} F_N$$

$$K = \frac{1}{2}mv^2$$
$$K = \frac{1}{2}I\omega^2$$
$$U = mgy \text{ (gravity)}$$
$$U = \frac{1}{2}kx^2 \text{ (spring)}$$

$$a = R\alpha$$
$$v = R\omega$$
$$\vec{L} = I\vec{\omega}$$
$$\vec{L} = \vec{r} \times \vec{p}$$
$$I = \sum_i m_i R_i^2$$

$$\vec{P}_0 = \vec{P}_f$$
$$\vec{L}_0 = \vec{L}_f$$
$$\Sigma p_{0x} = \Sigma p_{fx}$$
$$\Sigma p_{0y} = \Sigma p_{fy}$$

Question 1: A rod of length L and moment of inertia I is free to rotate about one end. Compare the torque required to rotate the rod with some fixed angular acceleration α when

(a) a mass m is attached $L/3$ away from the axis of rotation.

(b) a mass m is attached $2L/3$ away from the axis of rotation.

(c) a mass m is attached L away from the axis of rotation.

Question 2: When a woman on a frictionless rotating turntable extends her arms out horizontally, her angular momentum:

(a) must increase

(b) must decrease

(c) must remain the same

(d) may increase or decrease depending on her initial angular velocity

(e) tilts away from the vertical

Question 3: A 2.0-kg stone is tied to a 0.50-m long string and swung around a circle at a constant angular velocity of 12 rad/s. The net torque on the stone about the center of the circle is:

(a) 0

(b) 6.0 N m

(c) 12 N m

(d) 72 N m

(e) 140 N m

Question 4: A carousel has a 7-m radius and requires 8 s for a single revolution at full speed. A pig sits 3 m from the axis, and a horse sits 6 m from the axis.

(a) What is the period of a single revolution of the pig?

(b) Same, for the horse?

(c) What is the angular velocity of the pig?

(d) Same, for the horse?

(e) What is the velocity of the pig?

(f) Same, for the horse?

(g) What is the centripetal acceleration of the pig?

(h) Same, for the horse?

Question 5: Two pendulum bobs of unequal mass are suspended from the same fixed point by strings of equal length. The lighter bob is drawn aside and then released so that it collides with the other bob upon reaching the vertical position. The collision is elastic. What quantities are conserved during the collision?

(a) Both kinetic energy and momentum of the system.

(b) Only kinetic energy.

(c) Only momentum.

(d) Speed of lighter bob.

(e) None of the above.

Question 6: A circular disk of mass M and radius R_0 is at rest with its edge on the ground. A bullet of mass m hits and sticks to the disk near the top edge, causing it to roll. Assuming that momentum is conserved during the collision (that is, ignoring the friction from the ground), what is the linear and angular velocity of the disk after the impact? As a bonus, compare the kinetic energy before and after—is it conserved? (note: $I_{disk} = \frac{1}{2}MR_0^2$).

Question 7: Two large barges are moving in the same direction in still water. Barge 1 has a speed of 10 km/h and barge 2 has a speed of 20 km/h. While they are passing each other, coal is shoveled from barge 1 to barge 2 at a rate of 1000 kg/min. How much additional force must be applied by the engines so that neither ship changes speed? Do the calculation for both ships! Ignore friction, and assume the shoveling is perfectly sideways.

(a) Barge 1—

(b) Barge 2—

Extra Credit: Two rockets are racing from the sun to Venus. They travel in a straight line and both start from rest. Rocket A is twice as heavy as rocket B. If their accelerations are $a_A = g$ and $a_B = 2a_A = 2g$, find

(a) the center of mass of the two rockets as a function of time;

(b) the velocity of the center of mass as a function of time;

(c) the acceleration of the center of mass as a function of time.